ECE G287 Optical Detectors
Problem Set 2

1. Beginning from the definition that $p = \int_{-\infty}^{E_v} (1 - f(E))N(E)dE$ and assuming a parabolic isotropic hole band $E = (\hbar k)^2/2m^*$ show that

$$p = 2\left(\frac{2\pi m^* kT}{\hbar^2}\right)^{3/2} e^{-(E_f - E_v)/kT}$$

if it is assumed that $E_f - E >> kT$ for all the states in the valence band.

2. Find an expression for $N(E)$ (= number of electron states between E and $E+dE$) for the conduction band in silicon that has 6 pockets, each described by a longitudinal mass of m_ℓ=0.98 m_o and a transverse mass of m_t=0.19 m_o (m_o=free electron mass=9.11 × 10^{-31} kg.), so that:

$$E = \frac{(\hbar k_z)^2}{2m_\ell} + \frac{(\hbar k_x)^2}{2m_t} + \frac{(\hbar k_y)^2}{2m_t}$$

Then show that for $E_c - E_f >> kT$ the density of electrons in the conduction band can be given by

$$n = 2\left(\frac{2\pi m_d^* kT}{\hbar^2}\right)^{3/2} e^{-(E_c - E_f)/kT}$$

and find a value for m_d^*.

3. Consider a sample of silicon that has N_d=3 x 10^{17} cm^{-3} phosphorus donors located at $E_d = E_c$-44meV (44 meV below the conduction band edge). The energy gap is $E_c - E_v$ = 1.11 eV. Assume the electron density of states mass = 1.1 m_o and a hole density of states mass = 0.55 m_o and you can assume that $(E_c - E_f)$ and $(E_f - E_v) >> kT$. The number of electrons excited from the donor level is equal to the density of ionized donor states $N_d^+ = N_d(1 - f(E_d))$. By numerically solving for the condition that $n = N_d^+ + p$, find the position of the Fermi level E_f for T=4.2K, 77K, 300K, and 600K.. What percentage of the donor states are ionized at each temperature? Is the condition that $(E_c - E_f)$ and $(E_f - E_v) >> kT$ satisfied at each temperature.

4. Assuming an abrupt junction with N_d density of donors on the n-side and N_a density of acceptors on the p-side, derive expressions for the depletion region width $W(V_a)$ and the junction capacitance $C(V_a)$ as a function of applied voltage V_a.. Compare your expression for $C(V_a)$ with a simple expression that you would get assuming a simple parallel-plate capacitor with the depletion charge separated by $W(V_a)$.

5. Consider a two-dimensional electron gas in a GaAs heterostructure that consists of a single layer of GaAs of thickness *d* between two layers of GaAlAs. The electron energy bands of the conduction band electrons trapped in the "quantum well" between the two GaAlAs layers are given by:

$E = E_c + E_n + \frac{(\hbar k_x)^2}{2m^*} + \frac{(\hbar k_y)^2}{2m^*}$ where E_c is the conduction band edge and $E_n = \frac{\hbar^2}{2m^*}(n\pi/d)^2$ (*n* is any positive integer, *n*=1, 2, 3...).

Plot the energy of the bands for n=1, 2, and 3 vs. $k_t = \sqrt{k_x^2 + k_y^2}$ assuming that the thickness of the GaAs layer is *d*=150 Å and the effective mass of the conduction band in GaAs is $m^*=0.067\ m_o$. Plot the bands from $E=E_c$ to E_c+300 meV. (You can set the energy of the conduction band edge $E_c=0$.)

Find an expression for the density of states *N(E)* where *N(E)dE* is the number of electron states between *E* and *E + dE* and plot *N(E)* vs. 0<*E*<300 meV for the 150Å GaAs quantum well described above.